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THE MOTION OF A NONSYMMETRIC SELF-EXCITING GYROSTAT

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B. A. SMOL'NIKOV and M. V. STEPANOVA
(Leningrad)

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The motion of a self-exciting gyrostat is investigated in the special case where the force moment of the housing acts along one of the axes of inertia of the gyrostat while the projection of the gyrostatic moment (the moment of the relative momentum of the internal flywheels) on this axis is equal to zero. The parameters of the problem are the force moment and the moments of momenta of the flywheels, which are all assumed to be constant. The dependence of the hodograph of the angular velocity vector of the gyrostat on these parameters is investigated; the domains of parameter values corresponding to various types of motion are determined.

Grammel [1, 2] investigated a similar problem for a solid without internal rotations. The present study constitutes an extension of this familiar case.

1. The initial relations. The motion of a gyrostat with a constant gyrostatic moment $\mathbf{h}$ under the action of an external moment $\mathbf{m}$ is described by the following system of equations:

$$\begin{aligned} A_1\omega_1^{\cdot} + (A_3 - A_2)\,\omega_2\omega_3 + \omega_2 h_3 - \omega_3 h_2 &= m_1 \\ A_2\omega_2^{\cdot} - (A_3 - A_1)\,\omega_3\omega_1 + \omega_3 h_1 - \omega_1 h_3 &= m_2 \\ A_3\omega_3^{\cdot} + (A_2 - A_1)\,\omega_1\omega_2 + \omega_1 h_2 - \omega_2 h_1 &= m_3 \end{aligned} \qquad (1.1)$$

Here A_1, A_2, A_3 are the moments of inertia of the gyrostat with respect to its principal central axes x_1, x_2, x_3. For definiteness we assume that $A_1 < A_2 < A_3$; ω_1, ω_2, ω_3 are the projections of the angular velocity vector of the gyrostat on the axes x_1, x_2, x_3; h_1, h_2, h_3 are the projections of the gyrostatic moment, and m_1, m_2, m_3 are the projections of the external moment on the same axes.

The system of equations of gyrostat motion can be reduced to the form (1.1) when the absolute rather than the relative velocities of the flywheels are constant, i.e. when the flywheel shaft is not acted on by any moments [3].

Equations (1.1) are essentially nonlinear. Their exact solution for an arbitrary **m** cannot be constructed. However, a solution can be constructed if the vector **m** is directed along one of the principal axes of the gyrostat and if the projection of **h** on this axis is equal to zero. It is necessary to establish whether the moment **m** acts along the axis of the maximum (minimum) moment of inertia or along the axis of the intermediate moment of inertia. In the first two cases the equations of gyrostat motion are reducible to an identical form; in the latter case their form turns out to be somewhat different.

2. Action of the moment along the axis of maximum moment of inertia. Let the axis of action of the external moment **m** be x_3 (the axis of maximum moment of inertia of the housing), so that $h_3 = 0,\ m_1 = m_2 = 0.$

Now, introducing the variable

$$u = \left[\frac{(A_3 - A_1)(A_3 - A_2)}{A_1 A_2}\right]^{1/2} \int \omega_3 dt \tag{2.1}$$

we can write Eqs. (1.1) as

$$\begin{gathered}\frac{d\omega_1}{du} + \left[\frac{A_2 (A_3 - A_2)}{A_1 (A_3 - A_1)}\right]^{1/2} \omega_2 = h_2 \left[\frac{A_2}{A_1 (A_3 - A_1)(A_3 - A_2)}\right]^{1/2} \\ \frac{d\omega_2}{du} - \left[\frac{A_1 (A_3 - A_1)}{A_2 (A_3 - A_2)}\right]^{1/2} \omega_1 = - h_1 \left[\frac{A_1}{A_2 (A_3 - A_1)(A_3 - A_2)}\right]^{1/2} \\ A_3\left[\frac{A_1 A_2}{(A_3 - A_2)(A_3 - A_1)}\right]^{1/2} \frac{d^2u}{dt^2} + (A_2 - A_1)\,\omega_1\omega_2 + \omega_1 h_2 - \omega_2 h_1 = m_3\end{gathered} \tag{2.2}$$

The solution of the first two equations can be written as (2.3)

$$\omega_1 = - \Omega \left(\frac{A_3 - A_2}{A_1}\right)^{1/2} \sin u + \frac{h_1}{A_3 - A_1}, \qquad \omega_2 = \Omega \left(\frac{A_3 - A_1}{A_2}\right)^{1/2} \cos u + \frac{h_2}{A_3 - A_2}$$

Here Ω is the integration constant (the second integration constant is contained additively in u) ; we set $\Omega > 0$.

Let us introduce the dimensionless time τ by way of the relation

$$d\tau = \Omega \left[\frac{(A_2 - A_1)(A_3 - A_1)(A_3 - A_2)}{A_1 A_2 A_3}\right]^{1/2} dt \tag{2.4}$$

The last equation of system (2.2) can be written as

$$\frac{d^2u}{d\tau^2} - \sin u \cos u + \alpha \sin u + \beta \cos u = \varkappa \tag{2.5}$$

$$\begin{gathered}\alpha = - \frac{h_2}{\Omega} \frac{\sqrt{A_2 (A_3 - A_1)}}{(A_2 - A_1)(A_3 - A_2)}, \qquad \beta = - \frac{h_1}{\Omega} \frac{\sqrt{A_1 (A_3 - A_2)}}{(A_2 - A_1)(A_3 - A_1)} \\ \varkappa = - \frac{m_3}{\Omega^2 (A_2 - A_1)} \left[\frac{A_1 A_2}{(A_3 - A_1)(A_3 - A_2)}\right]^{1/2}\end{gathered} \tag{2.6}$$

We have thus reduced the problem to the investigation of nonlinear equation (2.5) which depends on the three parameters α, β and $\varkappa$.

Characterizing the motion of the gyrostat by the motion of the phase point in the space $\omega_1\ \omega_2\ \omega_3$, we see that the phase trajectories lie on the elliptic cylinder

$$\frac{[\omega_1 - h_1/(A_3 - A_1)]^2}{\Omega^2(A_3 - A_2)/A_1} + \frac{[\omega_2 - h_2/(A_3 - A_2)]^2}{\Omega^2(A_3 - A_1)/A_2} = 1 \tag{2.7}$$

so that the components ω_1 and ω_2 remain bounded. Let us consider the behavior of the component

$$\omega_3 = \Omega\left(\frac{A_2 - A_1}{A_3}\right)^{1/2}\frac{du}{d\tau} \tag{2.8}$$

Constructing the integral of Eq. (2.5),

$$\frac{1}{2}\left(\frac{du}{d\tau}\right)^2 - \frac{1}{4}\cos 2u + \alpha\cos u - \beta\sin u = \varkappa u + \gamma \tag{2.9}$$

(where γ is the integration constant), we obtain the following expression for ω_3:

$$\omega_3 = \Omega\sqrt{2(\gamma + \varkappa u + {}^1/_4\cos 2u - \alpha\cos u + \beta\sin u)(A_2 - A_1)/A_3} \tag{2.10}$$

Expression (2.10) together with (2.3) gives us a parametric relation for the phase trajectory in the space $\omega_1\omega_2\omega_3$. The dependence of u on τ is defined by the quadrature

$$\int\frac{du}{\sqrt{\gamma + \varkappa u + {}^1/_4\cos 2u - \alpha\cos u + \beta\sin u}} = \tau\sqrt{2} \tag{2.11}$$

which cannot be expressed in terms of known functions for $\varkappa \neq 0$.

To determine the character of the phase trajectory in the axes $\omega_1\omega_2\omega_3$ we need merely investigate the behavior of the phase curves described by Eq. (2.9) in the plane uu' ($u' = du\,/\,d\tau$) which is the development of elliptic cylinder (2.7). The structure of the phase picture on this plane clearly depends on the potential function in the integral of (2.9),

$$U(u) = -{}^1/_4\cos 2u + \alpha\cos u - \beta\sin u - \varkappa u \tag{2.12}$$

Depending on the values of the parameters α, β and $\varkappa$, the function $U(u)$ can have four, two, or zero extrema in the interval 2π. Their positions are given by the equation $dU\,/\,du = 0$; the positions of the points of merging of two extrema into an inflection point are given by the equation $d^2U(u)\,/\,du^2 = 0$. The set of equations

$$\sin u\cos u - \alpha\sin u - \beta\cos u = \varkappa \tag{2.13}$$
$$\cos 2u - \alpha\cos u + \beta\sin u = 0$$

therefore defines a surface in the parameter space α, β, $\varkappa$ which divides the latter into domains with differing numbers of extrema of the potential function $U(u)$.

Finding α and β from (2.13), we readily obtain the equation of this surface in parametric form,

$$\alpha = \cos^3 u - \varkappa\sin u, \qquad \beta = \sin^3 u - \varkappa\cos u \tag{2.14}$$

Expressing the cross sections of this surface with the planes $\varkappa = \mathrm{const}$ as the equations of a family of curves on the plane $\alpha\beta$, we see that for $\varkappa = 0$ the separation curve is an astroid,

$$\alpha^{2/3} + \beta^{2/3} = 1 \tag{2.15}$$

The values of α and β lying inside this astroid correspond to four real roots of the equation $dU\,/\,du = 0$; the values lying outside the astroid correspond to two roots. For $\varkappa \neq 0$ curve (2.14) clearly constitutes an equidistant of the astroid, i.e. a curve "parallel" to astroid (2.15); the quantity $|\varkappa|$ is numerically equal to the "distance" between the corresponding points of the astroid and the equidistant.

Figure 1 shows several curves defined by Eq. (2.14) for several values of $\varkappa < 0$. It is clear that as $|\varkappa|$ increases the vertices of the equidistant shift symmetrically with respect to the bisectrix of the first and third quadrants. These vertices, whose positions are given by the relations

$$d\alpha \,/\, du = d\beta \,/\, du = 0, \quad \sin u \cos u = -\,{}^1/_3\,\varkappa \tag{2.16}$$

likewise move along the astroid

$$\alpha - \beta = (1 + {}^2/_3\varkappa)^{3/2}, \qquad \alpha + \beta = (1 - {}^2/_3\varkappa)^{3/2} \tag{2.17}$$

as $\varkappa$ varies.

The existence of the vertices, i. e. of the singular points defined by (2.16), is possible only for $|\varkappa| < {}^3/_2$. For larger $|\varkappa|$ the equidistant of the astroid becomes a smooth closed curve which tends to a circle with the radius $|\varkappa|$ as $|\varkappa| \to \infty$. From Fig. 1 we see that as $|\varkappa|$ increases from 0 to ${}^1/_2$ the two opposite sides of the equidistant draw closer, making contact when $|\varkappa| = {}^1/_2$. Further increases in $|\varkappa|$ result in the appearance of a

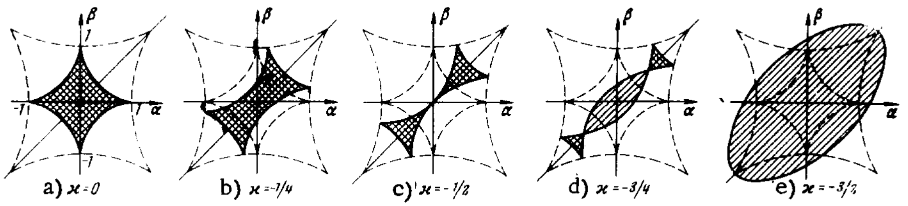

Fig. 1

new domain lying between the mutually intersecting aides of the equidistant, where (as we can show) the function $U\ (u)$ has no extrema whatever. The domain of four extrema then breaks down into two isolated subdomains whose extent diminishes with increasing $|\varkappa|$ and vanishes completely for $|\varkappa| = {}^3/_2$ In Fig. 1 the domain of four extrema is cross-hatched; the domain of two extrema is blank; the domain of absence of extrema is shaded. In the case $\varkappa > 0$ the "longitudinal" axis of the equidistant is the bisectrix of the second and fourth quadrants of the plane $\alpha\beta$.

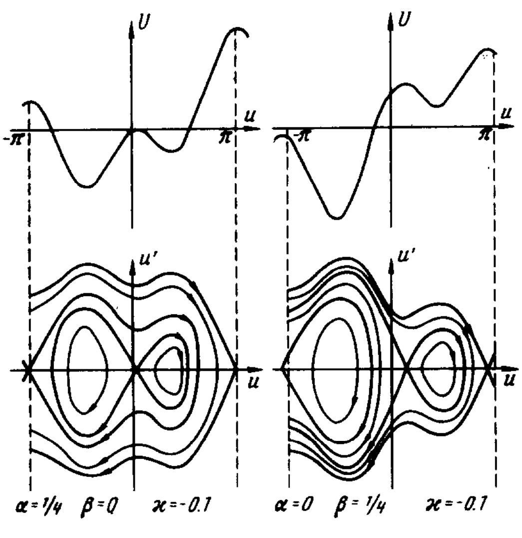

Fig. 2

Using the decomposition of the parameter space shown in Fig. 1, we can readily construct the phase trajectory picture in the plane uu' for any combination of values of the parameters α, β, $\varkappa$. Figure 2 shows sample phase portraits for two sets of parameter values; these portraits illustrate the existence of both closed and open phase trajectories as $|u|$ increases without limit.

In analyzing the motion of a gyrostat under the action of a restricted moment it is sometimes necessary to determine the character of variation of its sum kinetic moment **L**. Making use of (2.3) and (2.10), we readily obtain the following expression for this

$$L^2 = (A_1\omega_1 + h_1)^2 + (A_2\omega_2 + h_2)^2 + A_3^2\omega_3^2 = 2m_3uA_3\left[\frac{A_1A_2}{(A_3 - A_1)(A_3 - A_2)}\right]^{1/2} +$$

$$+ \Omega^2\left[A_1A_2 - \frac{A_3(A_2 - A_1)}{2} - 2\gamma A_3(A_2 - A_1)\right] + A_3^2\left[\frac{h_1^2}{(A_3 - A_1)^2} + \frac{h_2^2}{(A_3 - A_2)^2}\right] \quad (2.18)$$

We see that the square of the absolute value of the kinetic moment is a linear function of u(for $m_3 \neq 0$), and therefore attains its extrema together with u. The phase plane uu' thus enables us to establish the character of variation of the kinetic moment. Specifically, this plane enables us to determine when the kinetic moment $\mathbf{L}$ can have a secular component and when such a component does not exist.

3. Action of the moment along the axis of the intermediate moment of inertia. If we impose the conditions $h_2 = 0$, $m_1 = m_3 = 0$ and introduce the variable

$$v = \left[\frac{(A_2 - A_1)(A_3 - A_2)}{A_1A_3}\right]^{1/2}\int \omega_2\, dt \quad (3.1)$$

then the first and third equations of system (1.1) become

$$\frac{d\omega_1}{dv} + \left[\frac{A_3(A_3 - A_2)}{A_1(A_2 - A_1)}\right]^{1/2}\omega_3 = -h_3\left[\frac{A_3}{A_1(A_2 - A_1)(A_3 - A_2)}\right]^{1/2} \quad (3.2)$$

$$\frac{d\omega_3}{dv} + \left[\frac{A_1(A_2 - A_1)}{A_3(A_3 - A_2)}\right]^{1/2}\omega_1 = h_1\left[\frac{A_1}{A_3(A_2 - A_1)(A_3 - A_2)}\right]^{1/2}$$

In contrast to the preceding case the solution of this system can assume different forms for different initial conditions, namely (3.3)

$$\omega_1 = \Omega\left(\frac{A_3 - A_2}{A_1}\right)^{1/2}\frac{\operatorname{sh} v}{\operatorname{ch} v} + \frac{h_1}{A_2 - A_1}, \qquad \omega_3 = -\Omega\left(\frac{A_2 - A_1}{A_3}\right)^{1/2}\frac{\operatorname{ch} v}{\operatorname{sh} v} - \frac{h_3}{A_3 - A_2}$$

where either both upper or both lower particular solutions must be taken. Here Ω is an integration constant ($\Omega \gtrless 0$); the second integration constant is contained in v. In addition to general solutions (3.3), system (3.2) has two particular solutions not contained in (3.3), (3.4)

$$\omega_1 = \Omega\left(\frac{A_3 - A_2}{A_1}\right)^{1/2}e^{\pm v} + \frac{h_1}{A_2 - A_1}, \qquad \omega_3 = -\Omega\left(\frac{A_2 - A_1}{A_3}\right)^{1/2}e^{\pm v} - \frac{h_3}{A_3 - A_2}$$

In the plane $\omega_1\omega_3$ solutions (3.3) describe two families of hyperbolas with mutually perpendicular axes whose common asymptotes are given by particular solutions (3.4). The sign of the constant Ω defines motion along a branch of the corresponding hyperbola or asymptote. In contrast the preceding case, unbounded variation of v can be accompanied by unbounded variation of the components ω_1 and ω_3. To find the range of variation of v we transform the second equation of system (1.1) into

$$\frac{d^2v}{d\tau^2} + \operatorname{sh} v \operatorname{ch} v + \alpha\frac{\operatorname{sh} v}{\operatorname{ch} v} + \beta\frac{\operatorname{ch} v}{\operatorname{sh} v} = \varkappa \quad (3.5)$$

by converting to the dimensionless time τ in accordance with (2.4).

Here the upper functions correspond to the upper functions of solution (3.3); the lower functions correspond to the lower functions of the latter. The parameters α, β, $\varkappa$ are given by the formulas

$$\alpha = \frac{h_3\sqrt{A_3(A_2 - A_1)}}{\Omega(A_3 - A_1)(A_3 - A_2)}, \qquad \beta = \frac{h_1\sqrt{A_1(A_3 - A_2)}}{\Omega(A_3 - A_1)(A_2 - A_1)} \quad (3.6)$$

$$\varkappa = \frac{m_2}{\Omega^2 (A_3 - A_1)} \left[\frac{A_1 A_3}{(A_2 - A_1)(A_3 - A_2)} \right]^{1/2} \qquad \text{(cont.)}$$

Particular solutions (3.4) are associated with the following form of the equation for v :

$$d^2v / d\tau^2 \pm e^{\pm 2v} + (\alpha \pm \beta)e^{\pm v} = \varkappa \tag{3.7}$$

The signs in this equation must be chosen in accordance with the form of particular solution (3.4).

We shall consider only the case where the upper expressions are used in Eq. (3.5); the second case follows by formal transposition of the parameters α and β.

Writing out the first integral of Eqs. (3.5) and (3.7) and making use of (3.1), we obtain

$$\omega_2 = \Omega \sqrt{2(\gamma + \varkappa v - {}^1/_4 \operatorname{ch} 2v - \alpha \operatorname{ch} v - \beta \operatorname{sh} v)(A_3 - A_1) / A_2} \tag{3.8}$$

$$\omega_2 = \Omega \sqrt{2[\gamma + \varkappa v - {}^1/_2 e^{\pm 2v} \mp (\alpha \pm \beta) e^{\pm v}](A_3 - A_1) / A_2} \tag{3.9}$$

respectively.

Making use of these expressions, we can readily show that the square of the absolute value of the gyrostat moment L is a linear function of v as in the preceding case.

The behavior of the phase trajectories in the axes vv' is determined by the structure of the potential function $U(v)$ (contained in the radicands of (3.8) and (3.9)). In contrast to the preceding case, there can be either one or three extrema of the potential function in the interval $-\infty \leqslant v \leqslant \infty$. The boundary curve on the parameter plane $\alpha\beta$ can be determined from the conditions $dU / dv = 0$, $d^2U / dv^2 = 0$, which gives us

$$\alpha = -\operatorname{ch}^3 v - \varkappa \operatorname{sh} v, \qquad \beta = \operatorname{sh}^3 v + \varkappa \operatorname{ch} v \tag{3.10}$$

for Eq. (3.5).

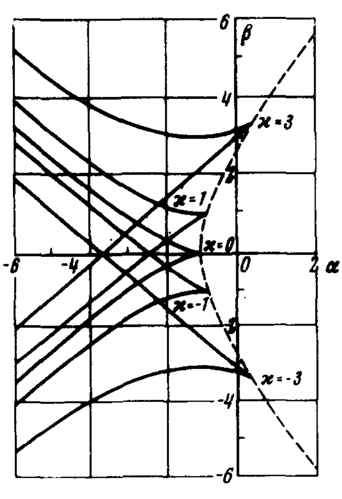

Fig. 3

The general shape of this curve for several values of $\varkappa$ is shown in Fig. 3. For $\varkappa = 0$ we have an explicit expression for the boundary curve,

$$\alpha^{2/3} - \beta^{2/3} = 1 \tag{3.11}$$

which can be called a "hyperbolic astroid". The latter has two isolated branches (as does an ordinary hyperbola); only one of its branches (the left-hand branch) satisfies Eqs. (3.10). As we see from the figure, this curve has a cusp at its vertex. For $\varkappa \neq 0$ the vertex, whose coordinates are easy to determine from the conditions similar to (2.17),

$$\operatorname{sh} 2v = -{}^2/_3 \varkappa \tag{3.12}$$

begins to move along the curve

$$3(\alpha^2 + \beta^2)^{2/3} = 4 + \beta^2 - \alpha^2 \tag{3.13}$$

(the broken curve in Fig. 3). The domain of parameters lying inside the branch of the hyperbolic astroid is associated with three extrema of the potential function; the domain outside this branch is associated with one extremum.

The potential function for particular solutions (3.4) is of the form

$$U(v) = {}^1/_2 e^{\pm 2v} \pm (\alpha + \beta)e^{\pm v} + \varkappa v \tag{3.14}$$

and its extrema are defined by the relation

$$e^{\pm v} = \pm {}^1/_2 (\alpha \pm \beta) \pm \sqrt{{}^1/_4 (\alpha \pm \beta)^2 \pm \varkappa} \tag{3.15}$$

where the double sign in front of the radical sign has the usual meaning, and where the remaining double signs refer to one or the other particular solution (3.3). The values of the parameters α, β, $\varkappa$ for which function (3.15) has two, one, or zero extrema can be readily determined from (3.15).

Comparison of the two above cases of gyrostat motion indicates that when the moment **m** acts along the axis of the maximum or minimum moment of inertia of the gyrostat, u (and therefore **L**) can increase without limit. On the other hand, when the moment acts along the axis of the intermediate moment of inertia **L** cannot, as a rule, increase without limit. Physically, this has to do with the fact that the rotation of a nonsymmetric solid about the axis of the intermediate moment of inertia is not stable. This makes the accumulation of kinetic energy along one of the axes (the axis x_2) impossible, and this energy is exchanged among the motions along all three axes.

4. The case of axial dynamic symmetry of a gyrostat. If the gyrostat housing has axial dynamic symmetry, then the above relations do not hold and the problem must be investigated separately. Two cases are possible, namely:

a) the moment **m** acts along the axis of dynamic symmetry;

b) the moment **m** acts perpendicularly to this axis.

In the first of these cases we set $h_3 = 0, m_1 = m_2 = 0,\ A_1 = A_2$ in Eqs. (1.1) (the axis of symmetry is x_3) and introduce the notation

$$u = \varepsilon \int \omega_3 dt, \qquad \varepsilon = (A_3 - A_1) / A_1 \tag{4.1}$$

to obtain

$$\omega_1 = \Omega \cos u + h_1 / (A_3 - A_1), \quad \omega_2 = \Omega \sin u + h_2 / (A_3 - A_1) \tag{4.2}$$

$$(A_3 / \varepsilon) d^2u / dt^2 - \Omega h_1 \sin u - \Omega h_2 \cos u = m_3 \tag{4.3}$$

We infer from this that the phase point in the space $\omega_1\omega_2\omega_3$ moves along the side surface of a circular cylinder parallel to the axis ω_3; the character of the motion of the point along this surface is defined by the familiar equation of a mathematical pendulum with a constant moment at the axis (Eq. (4.3)). Equilibrium positions in this case are possible only for

$$|m_3| \leqslant \Omega \sqrt{h_1^2 + h_2^2} \tag{4.4}$$

Other relations result in the second case, i.e. when we set $h_1 = 0,\ m_2 = m_3 = 0,$ $A_1 = A_2$ in equations of motion (1.1).

Introducing the notation

$$\varepsilon \int \omega_1 dt = v + D \tag{4.5}$$

where D is some still arbitrary constant, we can write the expressions for ω_2 and ω_3 in the form

$$\omega_3 = -h_2 (v + D) / \varepsilon A_3 + M \tag{4.6}$$

$$\omega_2 = -h_2 (v + D)^2 / 2\varepsilon A_3 + (v + D)(M + h_3 / \varepsilon A_1) + N$$

Here M and N are integration constants. The first equation of (1.1) yields the following equation for v:

$$\frac{d^2v}{dt^2} + \frac{h_2^2(v+D)^3}{2A_3^2} - \frac{3\varepsilon h_2 (v+D)^2}{2A_3}\left(M + \frac{h_3}{\varepsilon A_1}\right) + (v+D)\left[\varepsilon^2\left(M + \frac{h_3}{\varepsilon A_1}\right)^2 - \right.$$
$$\left. - \frac{\varepsilon h_2}{A_3}\left(N - \frac{h_2}{\varepsilon A_1}\right)\right] + \varepsilon^2 N\left(M + \frac{h_3}{\varepsilon A_1}\right) - \varepsilon\frac{h_2 M}{A_1} = \varepsilon m_1 \qquad (4.7)$$

Converting to dimensionless time,

$$d\tau = h_2 / A_3 \sqrt{2}\, dt \qquad (4.8)$$

and choosing the constant D in such a way that the coefficient of v^2 in the cubic polynomial in Eq. (4.7) vanishes, i.e. setting

$$D = -A_3 (M + h_3 / \varepsilon A_1) / h_2$$

we obtain

$$d^2v / d\tau^2 + v^3 - 3\lambda v = 2\delta \qquad (4.9)$$

Here

$$3\lambda = \left(M + \frac{h_3}{\varepsilon A_1}\right)\frac{\varepsilon^2 A_3^2}{h_2^2} + 2\left(N - \frac{h_2}{\varepsilon A_1}\right)\frac{\varepsilon A_3}{h_2}, \quad \delta = \frac{m_1 \varepsilon A_3^2}{h_2^2} - \frac{h_3 A_3^2}{h_2 A_1^2} \qquad (4.10)$$

We see that the parameter δ does not depend on the initial conditions; the parameter is determined by the initial values of ω_2 and ω_3. Equation (4.9) has the integral

$$^1/_2\,(dv / d\tau)^2 + {}^1/_4 v^4 - {}^3/_2\,\lambda v^2 = 2\delta v + \text{const} \qquad (4.11)$$

which enables us readily to express $\omega_1(v)$, and also to investigate the character of the phase trajectories on the plane vv'. This plane can be regarded as the development of the side surface of the parabolic cylinder defined by expressions (4.6) in the space $\omega_1\,\omega_2\,\omega_3$. The number of equilibrium points on this plane is either one or three. Proceeding as before, we can readily decompose the parameter plane $\lambda\delta$ into domains with one and three equilibrium states. The curve separating these domains is the semicubic parabola

$$\delta = (\lambda)^{3/2} \qquad (4.12)$$

Qualitatively, this case is close to the case of motion of a nonsymmetrical gyrostat analyzed in Sect. 3.

The solutions constructed above can be used to analyze the process of liquidation of the longitudinal component of the angular velocity of a gyrostat with relay switching of the sign of the controlling moment m in accordance with the sign of this component.

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